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Predictive Model of Inverted Neutrino Mass Hierarchy and Resonant Leptogenesis

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Abstract

We present a new realization of inverted neutrino mass hierarchy based on $S_3 \times \mathcal{U}(1)$ flavor symmetry. In this scenario, the deviation of the solar oscillation angle from $\pi/4$ is correlated with the value of θ_{13}, as they are both induced by a common mixing angle in the charged lepton sector. We find several interesting predictions: $\theta_{13} \geq 0.13$, $\sin^2\theta_{12} \geq 0.31$, $\sin^2\theta_{23} \simeq 0.5$ and $0 \leq \cos\delta \leq 0.7$ for the neutrino oscillation parameters and 0.01 eV $\lesssim m_{\beta\beta} \lesssim$ 0.02 eV for the effective neutrino mass in neutrino-less double β-decay. We show that our scenario can also explain naturally the observed baryon asymmetry of the universe via resonant leptogenesis. The masses of the decaying right–handed neutrinos can be in the range $(10^3 - 10^7)$ GeV, which would avoid the generic gravitino problem of supersymmetric models.

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1 Introduction

A lot has been learned about the pattern of neutrino masses and mixings over the past decade from atmospheric [1] and solar [2, 3] neutrino oscillation experiments. When these impressive results are supplemented by results from reactor [3]-[6] and accelerator [7] neutrino oscillation experiments, a comprehensive picture for neutrino masses begins to emerge. A global analysis of these results gives rather precise determination of some of the oscillation parameters [8]-[11]:

$$
\begin{gathered}
|\Delta m^2_{\rm atm}| = 2.4\cdot\left(1^{+0.21}_{-0.26}\right)\times 10^{-3}~{\rm eV}^2~, \qquad \sin^2\theta_{23} = 0.44\cdot\left(1^{+0.41}_{-0.22}\right)~, \\
\Delta m^2_{\rm sol} = 7.92\cdot(1\pm 0.09)\times 10^{-5}~{\rm eV}^2~, \qquad \sin^2\theta_{12} = 0.314\cdot\left(1^{+0.18}_{-0.15}\right)~, \\
\theta_{13} \lesssim 0.2~. \qquad (1)
\end{gathered}
$$

While these results are impressive, there are still many important unanswered questions. One issue is the sign of $\Delta m^2_{\rm atm} = m_3^2 - m_2^2$ which is presently unknown. This is directly linked to nature of neutrino mass hierarchy. A positive sign of $\Delta m^2_{\rm atm}$ would indicate normal hierarchy ($m_1 < m_2 < m_3$) while a negative sign would correspond to an inverted mass hierarchy ($m_2 \gtrsim m_1 > m_3$). Another issue is the value of the leptonic mixing angle θ_{13}, which currently is only bounded from above. A third issue is whether CP is violated in neutrino oscillations, which is possible (with $\theta_{13} \neq 0$) if the phase angle δ in the MNS matrix is non–zero. Forthcoming long baseline experiments [7], NOνA [12], T2K [13, 14] and reactor experiments double CHOOZ and Daya Bay will explore some or all these fundamental questions. Answers to these have the potential for revealing the underlying symmetries of nature.

While there exists in the literature a large number of theoretical models for normal neutrino mass hierarchy, such is not the case with inverted hierarchy. A large number of models for inverted hierarchy based on symmetries [15]-[23] that were proposed a few years ago are now excluded by the solar and Kamland data, which proved that θ_{12} is significantly away from the maximal value of $\pi/4$ predicted by most of these models. As a result, there is a dearth of viable inverted neutrino mass hierarchy models. In this paper, we attempt to take a step towards remedying this situation.

Here we suggest a class of models for inverted neutrino mass hierarchy based on $S_3 \times \mathcal{U}(1)$ flavor symmetry. S_3 is the non-Abelian group generated by the permutation of three objects, while the $\mathcal{U}(1)$ is used for explaining the mass hierarchy of the leptons. This $\mathcal{U}(1)$ symmetry is naturally identified with the anomalous $\mathcal{U}(1)$ of string origin. In our construction, the S_3 permutation symmetry is broken down to an approximate S_2 in the neutrino sector, whereas it is broken completely in the charged lepton sector. Such a setup enables us to realize effectively a $\nu_\mu \leftrightarrow \nu_\tau$ interchange symmetry in the neutrino sector (desirable for maximal mixing in atmospheric neutrino oscillations), while having non-degenerate charged leptons. The $\mathcal{U}(1)$ symmetry acts as leptonic $L_e - L_\mu - L_\tau$ symmetry, which is also desirable for an inverted neutrino mass spectrum. The breaking of S_2 symmetry in the charged lepton sector enables us to obtain θ_{12} significantly different from $\pi/4$.

Interestingly, we find that the amount of deviation of θ_{12} from $\pi/4$ is determined by θ_{13} through the relation

$$\sin^2\theta_{12} \simeq \frac{1}{2} - \tan\theta_{13}\cos\delta\ . \tag{2}$$

When compared with the neutrino data, the relation (2) implies the constraints (see Fig. 1):

$$\theta_{13} \geq 0.13\ , \qquad |\delta| \leq 0.75\ \ (\simeq 43^o)\ . \tag{3}$$

At the same time, the model gives

$$\sin^2\theta_{23} \simeq \frac{1}{2}(1-\tan^2\theta_{13})\ , \tag{4}$$

which is very close to 1/2. These predictions will be tested in forthcoming experiments. Somewhat similar relations have been obtained in scenarios with 'quark-lepton complementarity' [27]-[29] by postulating the relations $\theta_{12}+\theta_c \approx \pi/4$, $\theta_{23}+V_{cb} \approx \pi/4$ (θ_c is the Cabibbo angle). In our approach the leptonic mixing angles are inter-related by symmetries without involving the quark sector. Furthermore, we are able to derive the relations (2)-(4) from flavor symmetries (see section 4).

Our models have the right ingredients to generate the observed baryon asymmetry of the universe via resonant leptogenesis. The $\mathcal{U}(1)$ symmetry which acts on leptons as $L_e-L_\mu-L_\tau$ symmetry guarantee that two right–handed neutrinos that we use for see saw mechanism are quasi-degenerate. This feature leads to a resonant enhancement in the leptonic CP asymmetry, which in turn admits low right–handed neutrino masses, as low as few TeV. With such light right–handed neutrinos (RHN) generating lepton asymmetry, there is no cosmological gravitino problem when these models are supersymmetrized.

The class of neutrino mass models and leptogenesis scenario that we present here will work well in both supersymmetric and non-supersymmetric contexts. However, since low energy SUSY has strong phenomenological and theoretical motivations, we shall adopt the supersymmetric framework for our explicit constructions.

2 Predictive Framework for Neutrino Masses and Mixings

In order to build inverted hierarchical neutrino mass matrices which are predictive and which lead to successful neutrino oscillations, it is enough to introduce two right–handed neutrino states $N_{1,2}$. Then the superpotential relevant for neutrino masses is

$$W_\nu = l^T Y_\nu N h_u - \frac{1}{2}N^T M_N N\ , \tag{5}$$

where h_u denotes the up–type Higgs doublet superfield, while Y_ν and M_N are 3×2 Dirac Yukawa matrix and 2×2 Majorana mass matrix respectively. Their structures can be completely determined by flavor symmetries. In order to have predictive models of inverted hierarchy, the $L_e-L_\mu-L_\tau \equiv \mathbf{L}$

symmetry can be used [15]-[26]. This symmetry naturally gives rise to large θ_{23} and maximal θ_{12} angles. At the same time, the mixing angle θ_{13} will be zero. In order to accommodate the solar neutrino oscillations, the **L**-symmetry must be broken. The pattern of **L**-symmetry breaking will determine the relations and predictions for neutrino masses and mixings. As a starting point, in the neutrino sector let us impose $\mu - \tau$ interchange symmetry S_2: $l_2 \leftrightarrow l_3$, which will lead to maximal $\nu_\mu - \nu_\tau$ mixing, consistent with atmospheric neutrino data.

The leptonic mixing angles will receive contributions from both the neutrino sector and the charged lepton sector. As an initial attempt let us assume that the charged lepton mass matrix is diagonal. We will elaborate on altering this assumption in the next subsection.

For completeness, we will start with general couplings respecting the S_2 symmetry. Therefore, we have

$$Y_\nu = \begin{array}{c c} & \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{array}{c} l_1 \\ l_2 \\ l_3 \end{array} & \begin{pmatrix} \alpha & 0 \\ \beta' & \beta \\ \beta' & \beta \end{pmatrix} \end{array} , \qquad M_N = \begin{array}{c c} & \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{array}{c} N_1 \\ N_2 \end{array} & \begin{pmatrix} -\delta_N & 1 \\ 1 & -\delta_N^{'} \end{pmatrix} \end{array} M \ . \tag{6}$$

Note that setting $(1,2)$ element of Y_ν to zero can be done without loss of generality - by a proper redefinition of $N_{1,2}$ states. The couplings α, β and $(1,2), (2,1)$ entries in M_N respect **L** symmetry, while the couplings β', δ_N and $\delta_N^{'}$ violate it. Therefore, it is natural to assume that $|\beta'| \ll |\alpha|, |\beta|$, and $|\delta_N|, |\delta_N^{'}| \ll 1$. Furthermore, by proper field redefinitions all couplings in Y_ν can be taken to be real. Upon these redefinitions δ_N and $\delta_N^{'}$ entries in M_N will be complex.

Integration of the heavy $N_{1,2}$ states leads to the following 3×3 light neutrino mass matrix:

$$m_\nu = \begin{pmatrix} 2\delta_\nu^{'} & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & \delta_\nu & \delta_\nu \\ \sqrt{2} & \delta_\nu & \delta_\nu \end{pmatrix} \frac{m}{2} \ , \tag{7}$$

where

$$m = \frac{\langle h_u^0 \rangle^2}{M(1 - \delta_N \delta_N^{'})} \sqrt{2}\alpha \left(\beta + \beta' \delta_N^{'}\right) \ ,$$

$$\delta_\nu = \frac{\sqrt{2}}{\alpha} \frac{2\beta\beta' + \beta^2 \delta_N + (\beta')^2 \delta_N^{'}}{\beta + \beta' \delta_N^{'}} \ , \qquad \delta_\nu^{'} = \frac{\alpha}{\sqrt{2}} \frac{\delta_N^{'}}{\beta + \beta' \delta_N^{'}} \ . \tag{8}$$

The entries δ_ν, $\delta_\nu^{'}$ in (7) are proportional to the **L**-symmetry breaking couplings and therefore one naturally expects $|\delta_\nu|, |\delta_\nu^{'}| \ll 1$. These small entries are responsible for $\Delta m^2_{\rm sol} \neq 0$, i.e. for the solar neutrino oscillation. The neutrino mass matrix is diagonalized by unitary transformation

$U_\nu^T m_\nu U_\nu = \mathrm{Diag}\,(m_1, m_2, 0)$, were $U_\nu = U_{23}U_{12}$ with

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \qquad U_{12} \simeq \begin{pmatrix} \bar{c} & -\,\bar{s}e^{i\rho} & 0 \\ \bar{s}e^{-i\rho} & \bar{c} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{9}$$

where $\bar{c} = \cos\bar{\theta}$, $\bar{s} = \sin\bar{\theta}$ and

$$\tan\bar{\theta} \simeq 1 \pm \frac{1}{2}\kappa \ , \qquad \kappa = \frac{|\delta_\nu|^2 - |\delta_\nu^{'}|^2}{|\delta_\nu^* + \delta_\nu^{'}|} \ . \tag{10}$$

The phase ρ is determined from the relation

$$|\delta_\nu|\sin(\omega_\nu - \rho) = |\delta_\nu^{'}|\sin(\omega_\nu^{'} + \rho) \ , \quad \omega_\nu = \mathrm{Arg}(\delta_\nu) \ , \quad \omega_\nu^{'} = \mathrm{Arg}(\delta_\nu^{'}) \ , \tag{11}$$

and should be taken such that

$$|\delta_\nu|\cos(\omega_\nu - \rho) + |\delta_\nu^{'}|\cos(\omega_\nu^{'} + \rho) < 0 \ . \tag{12}$$

This condition ensures $\Delta m^2_{\mathrm{sol}} = m_2^2 - m_1^2 > 0$ needed for solar neutrino oscillations. For $\Delta m^2_{\mathrm{atm}}$ and the ratio $\Delta m^2_{\mathrm{sol}}/|\Delta m^2_{\mathrm{atm}}|$ we get

$$|\Delta m^2_{\mathrm{atm}}| \simeq |m|^2 \ , \quad \frac{\Delta m^2_{\mathrm{sol}}}{|\Delta m^2_{\mathrm{atm}}|} \simeq -2\left(|\delta_\nu|\cos(\omega_\nu - \rho) + |\delta_\nu^{'}|\cos(\omega_\nu^{'} + \rho)\right) = 2\left|\delta_\nu^* + \delta_\nu^{'}\right| \ . \tag{13}$$

With no contribution from the charged lepton sector, the leptonic mixing matrix is U_ν. From (9), (10) for the solar mixing angle we will have $\sin^2\theta_{12} = \frac{1}{2} \pm \frac{\kappa}{4}$. In order to be compatible with experimental data one needs $\kappa \approx 0.7$. On the other hand with $|\delta_\nu| \sim |\delta_\nu^{'}|$ and no specific phase alignment from (13) we estimate $|\delta_\nu| \sim |\delta_\nu^{'}| \sim 10^{-2}$. Thus we get the expected value $\kappa \sim 10^{-2}$, but with the θ_{12} mixing angle nearly maximal, which is incompatible with experiments. This picture remains unchanged with the inclusion of renormalization group effects. Therefore, we learn that it is hard to accommodate the neutrino data in simple minded inverted hierarchical neutrino mass scenario. In order for the scenario to be compatible with the experimental data we need simultaneously

$$\left|\delta_\nu^* + \delta_\nu^{'}\right| = \frac{\Delta m^2_{\mathrm{sol}}}{2|\Delta m^2_{\mathrm{atm}}|} \simeq 0.016 \qquad \text{and} \qquad \frac{|\delta_\nu|^2 - |\delta_\nu^{'}|^2}{|\delta_\nu^* + \delta_\nu^{'}|} = \mp(0.52 - 0.92) \ . \tag{14}$$

Therefore, one combination of δ_ν and $\delta_\nu^{'}$ must be $\sim$ 50-times larger than the other. This is indeed unnatural and no explanation for these conditions is provided at this stage. To make this point more clear let's consider the case with $\delta_\nu = 0$. In this case from (13) we have $|\delta_\nu^{'}| \simeq 0.016$. Using this in (10) we obtain $\sin^2\theta_{12} \geq 0.496$, which is excluded by the solar neutrino data (1).

Summarizing, although the conditions in (14) can be satisfied, it remains a challenge to have a natural explanation of needed hierarchies. This is a shortcoming of the minimal scenario. Below we present a possible solution to this conundrum which looks attractive and maintains predictive power without fine tuning by making use of mixing in the charged lepton sector.

2.1 Improved θ_{12} with $\theta_{13} \neq 0$

Let us now include the charged lepton sector in our studies. The relevant superpotential is

$$W_e = l^T Y_E e^c h_d \ , \tag{15}$$

where Y_E is 3×3 matrix in the family space. In general, Y_E has off–diagonal entries. Being so, Y_E will induce contributions to the leptonic mixing matrix. We will use this contribution in order to fix the value of θ_{12} mixing angle. It is desirable to do this in such a way that some predictivity is maintained. As it turns out, the texture

$$Y_E = \begin{pmatrix} 0 & a' & 0 \\ a & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} , \tag{16}$$

gives interesting predictions. In the structure (16) there is only one irremovable complex phase and we leave it in (1,2) entry. Thus, we make the parametrization $a' = \lambda_\mu \theta_e e^{i\omega}$, while all the remaining entries can be taken to be real. Diagonalizing $Y_E Y_E^\dagger$, namely, $U_e Y_E Y_E^\dagger U_e^\dagger = \left(Y_E^{\rm diag}\right)^2$, it is easy to see that

$$U_e = \begin{pmatrix} c & s e^{i\omega} & 0 \\ -s e^{-i\omega} & c & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{17}$$

where $c \equiv \cos t$, $s \equiv \sin t$ and $\tan t = -\theta_e$. Finally, the leptonic mixing matrix takes the form

$$U^l = U_e^* U_\nu \ , \tag{18}$$

where $U_\nu = U_{23} U_{12}$ can be derived from Eq. (9). Therefore, for the corresponding mixing elements we get

$$U_{e3}^l = -\frac{s}{\sqrt{2}} e^{-i(\omega+\rho)} \ , \quad |U_{e2}^l| = \frac{1}{\sqrt{2}} \left| c - \frac{s}{\sqrt{2}} e^{-i(\omega+\rho)} \right| \ , \quad |U_{\mu 3}^l| = \frac{c}{\sqrt{2}} \ . \tag{19}$$

Comparing these with those written in the standard parametrization of $U_{\rm MNS}$ we obtain the relations

$$s_{13} = -\frac{s}{\sqrt{2}} \ , \qquad \omega + \rho = \delta + \pi \ , \tag{20}$$

$$s_{12} c_{13} = |U_{e2}^l| \ , \quad s_{23} c_{13} = |U_{\mu 3}^l| \ . \tag{21}$$

Using (20) and (19) in (21) we arrive at the following predictions:

$$\sin^2 \theta_{12} = \frac{1}{2} - \sqrt{1 - \tan^2 \theta_{13}} \tan \theta_{13} \cos \delta \ ,$$

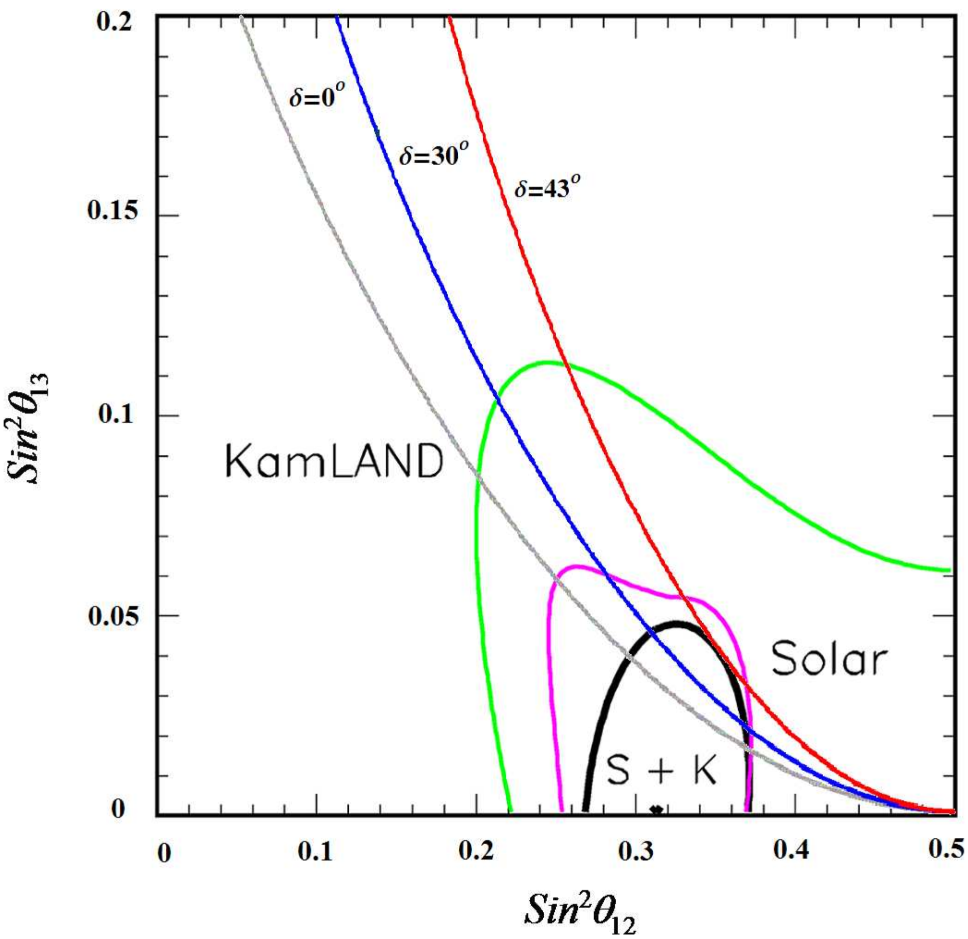

Figure 1: Correlation between θ_{12} and θ_{13} taken from Fogli *et al* of Ref. [11]. Three 'sloped' curves correspond to $\theta_{12} - \theta_{13}$ dependence (for three different absolute values of CP phase δ) obtained from our model according to Eq. (22).

$$\sin^2\theta_{23} = \frac{1}{2}\left(1 - \tan^2\theta_{13}\right) \ . \tag{22}$$

Since the CHOOZ results require $s_{13} \lesssim 0.2$, the first relation in (22), with the help of the solar neutrino data provides an upper bound for absolute value of the CP violating phase: $|\delta| \lesssim |\delta|_{\max} \approx 0.84 (\simeq 48^o)$. However, this estimate ignores the dependence of θ_{12} on the value of θ_{13} in the neutrino oscillation data. Having $\theta_{13} \neq 0$, this dependence shows up because one deals with three flavor oscillations. This has been analyzed in Ref. [11]. We show the results in Fig. 1 (borrowed from Ref. [11]) along with the constraints arising from our model. We have shown three curves corresponding to (22) for different values of $|\delta|$. Now we see that maximal allowed value for $|\delta|$ is $|\delta|_{\max} \simeq 0.75 (\simeq 43^o)$. Moreover, for a given δ we predict the allowed range for θ_{13}. In all cases the values are such that these relations can be tested in the near future. An interesting result from our

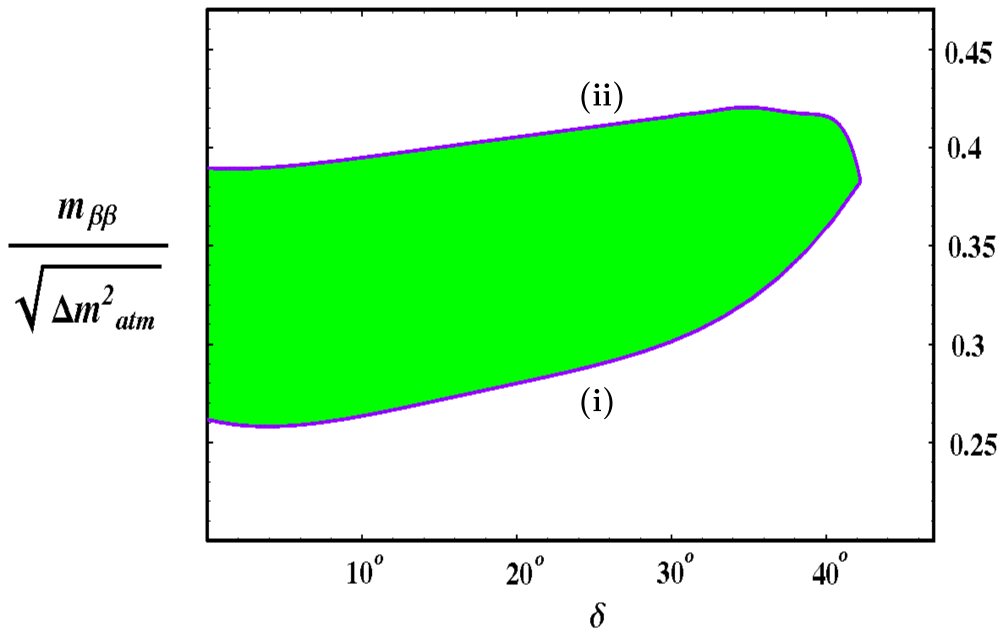

Figure 2: Curves (**i**) and (**ii**) respectively show the dependence of $\frac{m_{\beta\beta}}{\sqrt{\Delta m^2_{\rm atm}}}$'s low and upper bounds on absolute value of CP violating phase δ. The shaded region corresponds to values of $m_{\beta\beta}$ and $|\delta|$ realized within our model.

scenario is that we obtain lower and upper bounds for θ_{13} and $|\delta|$ respectively

$$\theta_{13} \geq 0.13 \ , \qquad\qquad |\delta| \leq 0.75 \ (\simeq 43^o) \ . \tag{23}$$

Finally, the neutrino-less double β-decay parameter in this scenario is given by

$$m_{\beta\beta} \simeq 2\sqrt{\Delta m^2_{\rm atm}} \tan\theta_{13} \frac{\sqrt{1-\tan^2\theta_{13}}}{\sqrt{1+\tan^2\theta_{13}}} \ . \tag{24}$$

We have neglected the small contribution (of order $\Delta m^2_{\rm solar}/\sqrt{\Delta m^2_{\rm atm}}$) arising from the neutrino mass matrix diagonalization. Since the value of θ_{13} is experimentally constrained ($\lesssim 0.2$), to a good approximation we have $m_{\beta\beta} \approx 2\sqrt{\Delta m^2_{\rm atm}} \tan\theta_{13}$. Using this result and the atmospheric neutrino data (1) we find $m_{\beta\beta} \lesssim 0.02$ eV. Knowledge of θ_{13}-dependence on $|\delta|$ (see Fig. 1) allows us to make more accurate estimates for the range of $m_{\beta\beta}$ for each given value of $|\delta|$. The dependence of $m_{\beta\beta}$ on $|\delta|$ is given in Fig. 2. We have produced this graph with the predictive relations (22), (24) using the neutrino data [11]. Combining these results we arrive at

$$0.011 \text{ eV} \lesssim m_{\beta\beta} \lesssim 0.022 \text{ eV}. \tag{25}$$

We see that the predicted range, depending on the value of $|\delta|$, is quite narrow. Future measurements of CP violating phase δ together with a discovery of the neutrino-less double β-decay will be another test for the inverted hierarchical scenario presented here.

3 Resonant Leptogenesis

Neutrino mass models with heavy right–handed neutrinos provide an attractive and natural framework for explaining the observed baryon asymmetry of the universe through thermal leptogenesis [30]. This mechanism takes advantage of the out-of-equilibrium decay of lightest right–handed neutrino(s) into leptons and the Higgs boson. In the scenario with hierarchical RHNs, a lower bound on the mass of decaying RHN has been derived: $M_{N_1} \geq 10^9$ GeV [31, 32] (under some not so unreasonable assumptions[4]). The reheating temperature cannot be much below the mass of N_1. In low energy SUSY models (with $m_{3/2} \sim 1$ TeV) this is in conflict with the upper bound on reheating temperature obtained from the gravitino abundance [34]-[41]. This conflict can be naturally avoided in the scenario of 'resonant leptogenesis' [42]-[44]. Due to the quasi-degeneracy in mass of the RHN states, the needed CP asymmetry can be generated even if the right–handed neutrino mass is lower than 10^9 GeV.

Our model of inverted hierarchical neutrinos involves two quasi-degenerate RHN states and has all the needed ingredients for successful resonant leptogenesis. This makes the scenario attractive from a cosmological viewpoint as well. Now we present a detailed study of the resonant leptogenesis phenomenon in our scenario.

The CP asymmetry is created by resonant out of equilibrium decays of N_1, N_2 and is given by [43, 44]

$$\epsilon_1 = \frac{\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}} \frac{(M_2^2 - M_1^2)\, M_1 \Gamma_2}{(M_2^2 - M_1^2)^2 + M_1^2 \Gamma_2^2} \ , \tag{26}$$

with a similar expression for ϵ_2. The asymmetries[5] ϵ_1 and ϵ_2 correspond to the decays of N_1 and N_2 respectively. Here M_1, M_2 are the mass eigenvalues of the matrix M_N in (6), while $\hat{Y}_\nu = Y_\nu U_N$ is the Dirac Yukawa matrix in a basis where RHN mass matrix is diagonal. The tree–level decay width of N_i is given as $\Gamma_i = (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{ii} M_i/(8\pi)$. The expression (26) deals with the regime $M_2 - M_1 \sim \Gamma_{1,2}/2$ (relevant for our studies) consistently and has the correct behavior in the limit $M_1 \to M_2$ [43, 44]. From (6) we have

$$U_N^T M_N U_N = \mathrm{Diag}\,(M_1, M_2) \ , \qquad U_N \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\,e^{ir} \\ e^{-ir} & 1 \end{pmatrix} \ , \tag{27}$$

[4] See ref. [33] for scenarios which violate this limit with hierarchical RHN masses.

[5] Here we use asymmetries averaged in relatively large time interval. The 'memory' effects [45] might cause changes in some cases.

with

$$M_2^2 - M_1^2 = 2M^2 \left|\delta_N^* + \delta_N^{'}\right| \ , \qquad \tan r = \frac{\mathrm{Im}\left(\delta_N - \delta_N^{'}\right)}{\mathrm{Re}\left(\delta_N + \delta_N^{'}\right)} \ . \tag{28}$$

Introducing the notations

$$\frac{\alpha}{\beta} = x \ , \qquad \frac{\beta'}{\beta} = x' \ , \tag{29}$$

we can write down the appropriate matrix elements needed for the calculation of leptonic asymmetry:

$$\begin{aligned}
(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} &= \frac{1}{2}\beta^2 \left(2 + x^2 + 2(x')^2 + 4xx' \cos r\right) \ , \\
(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} &= \frac{1}{2}\beta^2 \left(2 + x^2 + 2(x')^2 - 4xx' \cos r\right) \ , \\
\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2 &= -\frac{1}{4}\beta^4 \left(2 - x^2 - 2(x')^2 + 4xx' \cos r\right)^2 \sin 2r \ .
\end{aligned} \tag{30}$$

In terms of these entries the CP asymmetries are give by

$$\epsilon_1 = \frac{\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{21}^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}} \frac{|\delta_N^* + \delta_N^{'}|}{16\pi |\delta_N^* + \delta_N^{'}|^2 + (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}^2/(16\pi)} \ , \qquad \epsilon_2 = -\epsilon_1(1 \leftrightarrow 2) \ . \tag{31}$$

Since we have five independent parameters, in general one should evaluate the lepton asymmetry as a function of $x, x', |\delta_N|, |\delta_N^{'}|$ and r. Below we will demonstrate that resonant decays of $N_{1,2}$ can generate the needed CP asymmetry.

It turns out that for our purposes we will need $|\delta_N^* + \delta_N^{'}| \ll 1$. This, barring precise cancellation, implies $|\delta_N|, |\delta_N^{'}| \ll 1$. From the symmetry viewpoint and also from further studies, it turns out that $\left|\frac{x'}{x}\right| \ll 1$ is a self consistent condition. Taking this condition and the results from the neutrino sector into account, to a good approximation we have

$$\beta^2 = \frac{\sqrt{\Delta m_{\mathrm{atm}}^2} M}{\sqrt{2} x \langle h_u^0 \rangle^2} \ , \tag{32}$$

and

$$\begin{aligned}
\epsilon_1 \simeq \epsilon_2 \simeq \ & \frac{\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{12}^2}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}} \frac{|\delta_N^* + \delta_N^{'}|}{16\pi |\delta_N^* + \delta_N^{'}|^2 + (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}^2/(16\pi)} \simeq \\
& -\frac{(2 - x^2)^2}{2(2 + x^2)} \beta^2 \frac{|\delta_N^* + \delta_N^{'}|}{16\pi |\delta_N^* + \delta_N^{'}|^2 + (2 + x^2)^2 \beta^4/(64\pi)} \sin 2r \ ,
\end{aligned} \tag{33}$$

where in the last expression we have ignored x' contributions. This approximation is good for all practical purposes. The combination $|\delta_N^* + \delta_N^{'}|$ is a free parameter and since we are looking for a resonant regime, let us maximize the expression in (33) with respect to this variable. The maximal CP asymmetry is achieved with $|\delta_N^* + \delta_N^{'}| = (\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}/(16\pi)$. Plugging this value back in (33) and taking into account (30), (32) we arrive at

$$\bar{\epsilon}_1 \simeq \bar{\epsilon}_2 \simeq -\frac{(2 - x^2)^2}{2(2 + x^2)^2} \sin 2r \ , \tag{34}$$

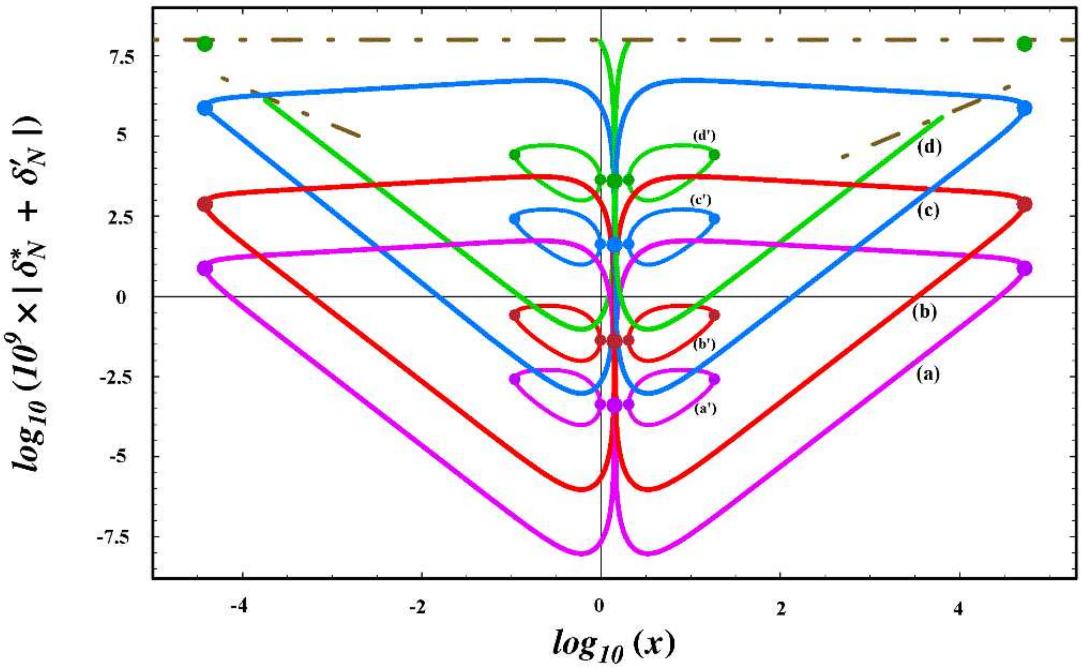

Figure 3: Resonant leptogenesis for inverted mass hierarchical neutrino scenario. In all cases $\frac{n_B}{s} = 9 \times 10^{-11}$ and $\tan\beta \simeq 2$. Curves **(a)**, **(b)**, **(c)**, **(d)** correspond respectively to the cases with $M = (10^4, 10^6, 10^9, 10^{11})$ GeV and $r = \pi/4$. The curves with primed labels correspond to same values of M, but with CP phase $r = 5 \cdot 10^{-5}$. Bold dots stand for a maximized values of CP asymmetry [see Eq. (38)]. The 'cut off' with horizontal dashed line reflects the requirement $\left|\delta_N^* + \delta_N'\right| \lesssim 0.1$. Two sloped dashed lines restrict low parts of the 'ovals' of $M = 10^{11}$ GeV, insuring the Yukawa coupling perturbativity.

where $\bar{\epsilon}_{1,2}$ indicate the maximized expressions, which do not depend on the scale of right–handed neutrinos. We can take these masses as low as TeV! The expression in (34) reaches the maximal values for $x \ll 1$ and $x \gg 1$. However, the final value of x will be fixed from the observed baryon asymmetry.

The lepton asymmetry is converted to the baryon asymmetry via sphaleron effects [46] and is given by $\frac{n_B}{s} \simeq -1.48 \cdot 10^{-3}(\kappa_f^{(1)}\epsilon_1 + \kappa_f^{(2)}\epsilon_2)$, where $\kappa_f^{(1,2)}$ are efficiency factors given approximately by [47]

$$\kappa_f^{(1,2)} = \left(\frac{3.3 \cdot 10^{-3}\ \text{eV}}{\tilde{m}_{1,2}} + \left(\frac{\tilde{m}_{1,2}}{0.55 \cdot 10^{-3}\ \text{eV}}\right)^{1.16}\right)^{-1} ,$$

$$\text{with} \quad \tilde{m}_1 = \frac{\langle h_u^0\rangle^2}{M_1}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11} \ , \quad \tilde{m}_2 = \frac{\langle h_u^0\rangle^2}{M_2}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22} \ . \qquad (35)$$

In our model, with $\left|\frac{x'}{x}\right| \ll 1$ we have

$$\tilde{m}_1 \simeq \tilde{m}_2 \simeq \frac{\sqrt{\Delta m^2_{\rm atm}}}{2\sqrt{2}x}(2+x^2) \simeq 0.017 \text{ eV} \times \frac{2+x^2}{x} \ . \tag{36}$$

This also gives $\kappa_f^{(1)} \simeq \kappa_f^{(2)} \equiv \kappa_f$ and as a result we obtain

$$\left.\frac{n_B}{s}\right|_{\epsilon=\bar{\epsilon}} \simeq 1.48 \cdot 10^{-3} \kappa_f(x) \frac{(2-x^2)^2}{(2+x^2)^2} \sin 2r \ . \tag{37}$$

With $\sin 2r = 1$ in order to reproduce the experimentally observed value $\left(\frac{n_B}{s}\right)^{\rm exp} = 9 \cdot 10^{-11}$ we have four possible choices of x: $x = 3.8 \cdot 10^{-5}$, $x = 5.3 \cdot 10^4$, $x = \sqrt{2} - 0.0047$ or $x = \sqrt{2} + 0.0047$. For these values of x we have respectively

$$\left|\delta_N^* + \delta_N^{'}\right|_{\epsilon=\bar{\epsilon}} \simeq \frac{2+x^2}{32\sqrt{2}\pi x} \frac{\sqrt{\Delta m^2_{\rm atm}} M}{\langle h_u^0 \rangle^2} \simeq$$

$$\left(6 \cdot 10^{-7} \ , \ 6 \cdot 10^{-7} \ , \ 3.2 \cdot 10^{-11} \ , \ 3.2 \cdot 10^{-11}\right) \times \frac{1+\tan^2\beta}{\tan^2\beta} \frac{M}{10^6 \text{GeV}} \tag{38}$$

(fixed from the condition of maximization). The MSSM parameter $\tan\beta$ should not be confused with Yukawa coupling in (32)). Note that these results are obtained at the resonant regime $|M_2 - M_1| = \Gamma_{1,2}/2$. If we are away from this point, then the baryon asymmetry will be more suppressed and we will need to take different values of x. In Fig. 3 we show $\left|\delta_N^* + \delta_N^{'}\right| - x$ dependence corresponding to baryon asymmetry of $9 \cdot 10^{-11}$. The curves are constructed with Eqs. (32), (33). We display different cases for different values of the mass M and for two values of CP violating phase r. For smaller values of r the 'ovals' shrink indicating that there is less room in $\left|\delta_N^* + \delta_N^{'}\right| - x$ plane for generating the needed baryon asymmetry. We have limited ourselves to $\left|\delta_N^* + \delta_N^{'}\right| \lesssim 0.1$. Above this value the degeneracy disappears and the validity of our expression (26) breaks down[6]. Also, in this regime the inverted mass hierarchical neutrino scenario becomes unnatural. The dashed horizontal line in Fig. 3 corresponds to this 'cut–off'. This limits the cases with larger masses [case (d) in Fig. 3, of $M = 10^{11}$ GeV]. The sloped dashed cut–off lines appear due to the requirement that the Yukawa couplings be perturbative ($\alpha, \beta \lesssim 1$). As one can see from (32), for sufficiently large values of M, with $x \gg 1$ or $x \ll 1$, one of the Yukawa couplings becomes non-perturbative.

As we see, in some cases (especially for suppressed values of r) the degeneracy in mass between N_1 and N_2 states is required to be very accurate, i.e. $\left|\delta_N^* + \delta_N^{'}\right| \ll 1$. In section 4 we discuss the possibility for explaining this based on symmetries.

[6]There will be another contributions to the CP asymmetry, the vertex diagram, which would be significant in the non-resonant case.

4 Model with $S_3 \times \mathcal{U}(1)$ Symmetry

In this section we present a concrete model which generates the needed textures for the charged lepton and the neutrino mass matrices. The Lagrangian of the model is the most general under the symmetries of the model. The model explains the hierarchies of the charged leptons, neutrinos and the lepton mixing angles. Therefore, the relations (22) are derived as a consequence of symmetries.

The model presented here also blends in well with the leptogenesis scenario investigated in the previous section. In particular, the splitting between the masses of nearly degenerate heavy neutrinos have the right magnitude needed for resonant leptogenesis.

We wish to have an understanding of the hierarchies and the needed zero entries in the Dirac and Majorana neutrino couplings. Also, the values of masses $M_{N_{1,2}} \simeq M \lesssim 10^8$ GeV and their tiny splitting must be explained. Note that one can replace $\mathbf{L} = L_e - L_\mu - L_\tau$ symmetry by other symmetry, which will give approximate $\mathbf{L}$. For this purpose the anomalous $\mathcal{U}(1)$ symmetry of string origin is a good candidate [20]-[22]. In our scenario the charged lepton sector also plays an important role. In particular, the structure (16) is crucial for the predictions presented in the previous sections. We wish to understand this structure also by symmetry principles. For this a non-Abelian discrete flavor symmetries can be very useful [54]-[63]. Therefore, in addition, we introduce S_3 permutation symmetry. S_3 will be broken in two steps: $S_3 \to S_2 \to 1$. Since in the neutrino sector we wish to have S_2 symmetry, we will arrange for that sector to feel only the first stage of breaking.

Thus, the model we present here is based on $S_3 \times \mathcal{U}(1)$ flavor symmetry. The S_3 permutation group has three irreducible representations $\mathbf{1}$, $\mathbf{1'}$ and $\mathbf{2}$, where $\mathbf{1'}$ is an odd singlet while $\mathbf{1}$ and $\mathbf{2}$ are true singlet and doublet respectively. With doublets denoted by two component vectors, it is useful to give the product rule

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1 y_1 + x_2 y_2)_{\mathbf{1}} \;\oplus\; (x_1 y_2 - x_2 y_1)_{\mathbf{1'}} \;\oplus\; \begin{pmatrix} x_1 y_2 + x_2 y_1 \\ x_1 y_1 - x_2 y_2 \end{pmatrix}_{\mathbf{2}} \tag{39}$$

where subscripts denote the representation of the corresponding combination. The other products are very simple. For instance $\mathbf{1} \times \mathbf{1} = \mathbf{1}$, $\mathbf{1'} \times \mathbf{1} = \mathbf{1'}$, etc.

As far as the $\mathcal{U}(1)$ symmetry is concerned, a superfield ϕ_i transforms as

$$\mathcal{U}(1): \quad \phi_i \to e^{iQ_i} \phi_i \ , \tag{40}$$

where Q_i is the $\mathcal{U}(1)$ charge of ϕ_i. The $\mathcal{U}(1)$ symmetry will turn out to be anomalous. Such an anomalous $U(1)$ factor can appear in effective field theories from string theory upon compactification to four dimensions. The apparent anomaly in this $\mathcal{U}(1)$ is canceled through the Green-Schwarz mechanism [48]. Due to the anomaly, a Fayet-Iliopoulos term $-\xi \int d^4\theta V_A$ is always generated

Table 1: Transformation properties under $S_3 \times \mathcal{U}(1)$, and Q_Z-charges of $\mathcal{Z}_4$ parity: $\phi_i \to e^{i\frac{\pi}{2}Q_Z}\phi_i$, $W \to -W$.

	$\vec{S}$	$\vec{T}$	Y	Z	e_1^c	$\vec{e}^{\,c}$	l_1	$\vec{l}$	N_1	N_2	h_u	h_d
S_3	**2**	**2**	**1**	**1**	**1**	**2**	**1**	**2**	**1**	**1**	**1**	**1**
$\mathcal{U}(1)$	0	-1	2	0	4	2	1	0	-1	k	0	-2
Q_Z	2	0	2	2	-1	-1	-1	-1	-1	1	2	2

[49, 50] and the corresponding D_A-term has the form [51]-[53]

$$\frac{g_A^2}{8}D_A^2 = \frac{g_A^2}{8}\left(-\xi + \sum Q_i|\phi_i|^2\right)^2 \ , \quad \xi = \frac{g_A^2 M_P^2}{192\pi^2}\mathrm{Tr}Q \ . \tag{41}$$

In SUSY limit one chiral superfield should acquire a VEV in order to set D_A-term to be zero.

For $S_3 \times \mathcal{U}(1)$ breaking we introduce the MSSM singlet scalar superfields $\vec{S}, \vec{T}, X$, Y and Z. (vector symbols will denote S_3 doublets). We also introduce a discrete $\mathcal{Z}_4$ R-symmetry under which the superfields transform as $\phi_i \to e^{i\frac{\pi}{2}Q_Z}\phi_i$ and the superpotential changes sign: $W \to -W$. The transformation properties - the S_3 'membership', $\mathcal{U}(1)$ and $\mathcal{Z}_4$ charges - of all involved superfields are given in Table 1.

Let us first discuss the symmetry breaking. The most general renormalizable 'scalar' superpotential consistent with symmetries has the form

$$W_{sc} = Y\vec{T}^2 + \frac{\lambda_1}{2}Z(\vec{S}^2 - \Lambda^2) + \frac{\lambda_2}{3}\vec{S}^3 + \frac{\lambda_3}{3}Z^3 \ . \tag{42}$$

From the F-flatness conditions $F_{\vec{S}} = F_{\vec{T}} = F_Z = 0$ we have the solutions

$$\langle\vec{S}\rangle = (0,\ V_S) \ , \qquad \text{with} \qquad V_S = \Lambda\left(1 + 2\lambda_2^2\lambda_3/\lambda_1^3\right)^{-1/2}$$

$$\langle Z\rangle = V_S\lambda_2/\lambda_1 \ , \quad \langle Y\rangle = 0 \ . \tag{43}$$

From $F_Y = 0$ we get the condition $\langle\vec{T}^2\rangle = T_1^2 + T_2^2 = 0$ which is satisfied by $\langle\vec{T}\rangle = V_T\cdot(1,\ i)$ with unfixed V_T from the superpotential. However, the non–zero value of V_T is insured from the cancelation of D_A-term of (41). Namely, with $\xi < 0$, we have $V_T = \sqrt{-\xi/2}$. Thus, all VEVs are fixed already in the unbroken SUSY limit and there are no flat directions. We need to make sure that VEV configurations remain also stable with inclusion of higher order operators. At the renormalizable level there are no couplings between $\vec{T}$ and $\vec{S}$. However, higher order interactions

between these states may change the winding of their VEVs. The lowest order operators of this type respecting all other symmetries are $\frac{1}{M_{\rm Pl}^2}Y\vec{T}^2(\vec{S}^2+\vec{S}Z+Z^2)$. In the presence of these couplings the $F_Y=0$ condition gets modified and we obtain

$$\langle\vec{T}\rangle=V_T\cdot(1,\ i(1+\eta))\ ,\quad \text{with}\quad \eta\sim\epsilon_s^2\ ,\qquad \text{where}\qquad \epsilon_s=V_S/M_{\rm Pl} \tag{44}$$

(with $\langle Z\rangle\sim V_S$). As we see, the winding of $\langle\vec{T}\rangle$ is slightly changed (with $\eta\ll 1$, i.e. $\epsilon_s\ll 1$). This change will not have any impact in the charged lepton sector and for the light neutrino masses and mixings. However, this will turn out to be important in shifting the right–handed neutrino mass degeneracy and therefore for resonant leptogenesis.

We will use the following parametrization

$$\frac{\langle Z\rangle}{M_{\rm Pl}}\sim\frac{V_S}{M_{\rm Pl}}\equiv\epsilon_s\ ,\qquad \frac{V_T}{M_{\rm Pl}}\equiv\epsilon\ . \tag{45}$$

These two parameters will be used in expressing hierarchies between masses and mixings of the leptons. All non-renormalizable operators that we consider in the charged lepton sector will be cut off by appropriate powers of the Planck scale $M_{\rm Pl}$ and therefore in those operators the powers of ϵ_s and ϵ will appear. There are also operators cut off with a different scale which can be obtained by integrating out some vector-like states with masses below $M_{\rm Pl}$.

Let us start with the charged lepton sector. For the tau lepton mass the operator

$$\frac{1}{\langle Z\rangle M_*}\left(\vec{l}\cdot\vec{S}\right)_{\mathbf{1}}\left(\vec{e}^{\ c}\cdot\vec{S}\right)_{\mathbf{1}}h_d\ , \tag{46}$$

is relevant, where S_3 contraction is in the singlet $\mathbf{1}$-channel. This operator can emerge by decoupling of heavy L, E^c states in $\mathbf{1}$ representation of S_3, as shown in the diagram of Fig. 4. Eq. (46) gives $\lambda_\tau\sim V_S/M_*$, where M_* is a mass of $E^c,\overline{E}^c$ states.

Next, we include the following Planck scale suppressed operators:

$$\frac{1}{M_{\rm Pl}}\vec{l}\cdot\vec{e}^{\ c}\cdot(\vec{S}+Z)h_d+\frac{1}{M_{\rm Pl}^2}l_1\vec{e}^{\ c}\cdot\vec{T}\cdot(\vec{S}+Z)h_d+\frac{1}{M_{\rm Pl}^3}e_1^c\vec{l}\cdot\vec{T}^2\cdot(\vec{S}+Z)h_d+\frac{1}{M_{\rm Pl}^4}l_1e_1^c\vec{T}^3\cdot(\vec{S}+Z)h_d\ . \tag{47}$$

Substituting appropriate VEVs in (46), (47) and taking into account that $\vec{l}=(l_2,\ l_3)$, $\vec{e}^{\ c}=(e_2^c,\ e_3^c)$, for the charged lepton Yukawa matrix we obtain

$$\begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array}\begin{array}{c} \begin{array}{ccc} e_1^c & e_2^c & e_3^c \end{array} \\ \begin{pmatrix} \epsilon_s\epsilon^3 & \epsilon_s\epsilon & \epsilon_s\epsilon \\ \epsilon_s\epsilon^2 & \epsilon_s & 0 \\ \epsilon_s\epsilon^2 & 0 & \lambda_\tau \end{pmatrix} \end{array}\ , \tag{48}$$

which nearly has the desired structure of (16). It is easy to see that the (1, 1) entry of (48) not presented in (16) does not change the predictive relations obtained in sect. 2.1. The (1, 3) and

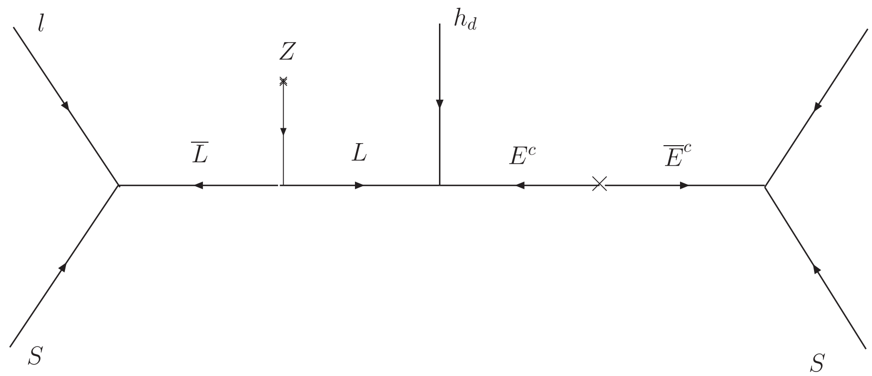

Figure 4: Diagram generating the operator of Eq. (46)

$(3,1)$ entries have no new parameters, and they shift relations in (16) by $\lesssim 1\%$. Indeed, from (48) we conclude that

$$\epsilon \simeq 0.13 - 0.2\ , \qquad \epsilon_s \sim \lambda_\tau \epsilon^2 \tag{49}$$

(this provides $\lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1$, which is compatible with the observed hierarchies). Therefore, the results of sect. 2.1 are robust.

Now we turn to the neutrino sector. With transformation properties given in Table 1, and for integer $k > 0$, the relevant couplings have the form

$$\begin{array}{c c} & \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{array}{c} l_1 \\ \vec{l} \end{array} & \left(\begin{array}{cc} \frac{Z}{M_{\rm Pl}} & \frac{\vec{T}^{k+1}}{M_{\rm Pl}^{k+1}} \\ 0 & \frac{\vec{T}^k}{M' M_{\rm Pl}^{k-1}} \end{array} \right) h_u\ , \end{array} \qquad \begin{array}{c c} & \begin{array}{cc} N_1 & N_2 \end{array} \\ \begin{array}{c} N_1 \\ N_2 \end{array} & \left(\begin{array}{cc} 0 & (Z+\vec{S})\frac{\vec{T}^{k-1}}{M_{\rm Pl}^k} \\ (Z+\vec{S})\frac{\vec{T}^{k-1}}{M_{\rm Pl}^k} & \frac{\vec{T}^{2k}}{M_{\rm Pl}^{2k}} \end{array} \right) M_R\ . \end{array} \tag{50}$$

M' is some cut off scale lower than $M_{\rm Pl}$. We have found one interesting example: for $k = 5$, and $\frac{V_T}{M'} \sim 1$ we obtain

$$Y_\nu = \begin{pmatrix} \epsilon_s & \epsilon^6 \\ 0 & \epsilon^4 \\ 0 & i\epsilon^4 \end{pmatrix}\ , \qquad M_N = \begin{pmatrix} 0 & \epsilon_s \epsilon^4 \\ \epsilon_s \epsilon^4 & \epsilon_s^2 \epsilon^{10} \end{pmatrix} M_R \tag{51}$$

(where we have used the property $\langle \vec{T} \rangle^{10}/M_{\rm Pl}^{10} \sim \epsilon_s^2 \epsilon^{10}$). Making a rotation of $N_{1,2}$ states to set (1,2) entry of the first matrix of (51) to zero and at the same time performing phase redefinitions we will

arrive at the form of (6) with

$$M = M_R \epsilon_s \epsilon^4 \sim M_R \epsilon^6 \lambda_\tau \ , \qquad \alpha \sim \epsilon_s \ , \quad \beta \sim \epsilon^4 \ , \quad \left|\delta_N^* + \delta_N^{'}\right| \sim \epsilon^6 \epsilon_s \sim \epsilon^8 \lambda_\tau \ , \tag{52}$$

and

$$\sqrt{|\Delta m^2_{\rm atm}|} \sim \frac{\langle h_u \rangle^2}{M_R} \ , \qquad \left|\delta_\nu^* + \delta_\nu^{'}\right| \sim \frac{\epsilon^2}{\sqrt{2}} \ . \tag{53}$$

Therefore, we get the right magnitude for $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$, while experimentally measured value of $|\Delta m^2_{\rm atm}|$ dictates $M_R = (10^{13} - 10^{14})$ GeV. Thus, from (52) we can estimate the absolute value of the RHN mass. For $\tan\beta \simeq 2$ (the value used for numerical studies in sect. 3) we get $M = (10^6 - 10^8)$ GeV. This range includes the values of RHN masses such that SUSY gravitino problem is avoided. At the same time we get $\left|\delta_N^* + \delta_N^{'}\right| \sim 10^{-9} - 5 \cdot 10^{-8}$ and $x \sim \lambda_\tau/\epsilon^2 \sim 1$. All these values work well for resonant leptogenesis (see Fig. 3). Therefore, we conclude that the model presented in this section works well for inverted neutrino mass hierarchical scenario and also insures the success of resonant leptogenesis.

Finally, we briefly comment on some aspects of low energy phenomenology of the presented model. The superpotential term $Y h_u h_d$ is allowed by symmetries of the model, which has a potential for generating the μ-term with $\langle Y \rangle \sim 1$ TeV induced after SUSY breaking. As far as the quark sector is concerned, the Yukawa couplings $q u^c h_u$ and $q d^c h_d$ are allowed with the following $\mathcal{U}(1)$ and $\mathcal{Z}_4$ charge assignment: $Q(q, u^c, d^c) = (y, -y, 2 - y)$ and $Q_Z(q, u^c, d^c) = (1, -1, 1)$. This charge assignment is flavor independent. However, if desired one can also select flavor dependent charges for understanding of hierarchies between quark masses and CKM mixings. The freedom in the selection of y can be exploited for the simultaneous cancelation of $SU(3)^2 \times \mathcal{U}(1)$ and $SU(2)^2 \times \mathcal{U}(1)$ mixed anomalies via Green-Schwarz mechanism (achieved with $y = 7/9$). Also, one can verify that $SU(3)^2 \times \mathcal{Z}_4$ and $SU(2)^2 \times \mathcal{Z}_4$ anomalis are automatically zero with the above $\mathcal{Z}_4$ charge assignment. Therefore, $\mathcal{Z}_4$ can be identified as a discrete gauge symmetry. One remarkable feature is that with the $\mathcal{Z}_4$ symmetry, matter parity is automatic. Indeed, with the Q_Z charge assignment, a Z_2 subgroup of $\mathcal{Z}_4$ R-symmetry remains unbroken. Therefore, $\mathcal{Z}_4$-symmetry insures that the model has realistic phenomenology with a stable LSP.

5 Conclusions

In this paper we have presented a new class of models which realizes an inverted spectrum for neutrino masses. These models predict a definite correlation between neutrino mixing angles θ_{12} and θ_{13}. Deviation of θ_{12} from $\pi/4$ is controlled by the value of θ_{13}. Our results are given in Eqs. (22)-(25) and plotted in Figs. 1, 2.

We have presented a concrete model based on an S_3 permutation symmetry augmented with a discrete $\mathcal{Z}_4$ R-symmetry and $\mathcal{U}(1)$ symmetry acting on the three flavors.

Our model can naturally lead to resonant leptogenesis since two right–handed neutrinos are quasi-degenerate. The predictions of our model are testable in forthcoming experiments.

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